

Towards a GOLEM NLO prediction for $pp \rightarrow bbbb$ at LHC:

virtual corrections to the quark induced part.

Thomas Binotto



In collaboration with: A. Guffanti, J.Ph. Guillet, T. Reiter, J. Reuter

October 9, 2008
HP2 workshop
Buenos Aires, Argentina

Content:

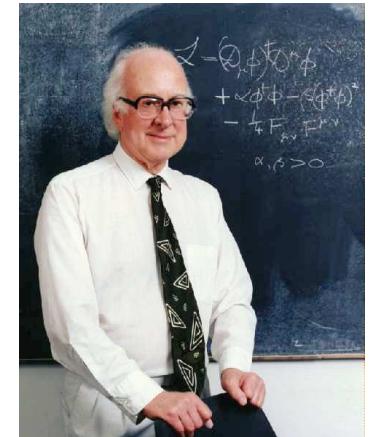
- Motivation: LHC @ NLO
- Framework for one-loop amplitudes: the GOLEM project
- The process $qq \rightarrow b\bar{b}b\bar{b}$
- Summary

The advent of the LHC era

LHC: Large Hadron Collider at CERN, $\sqrt{s} = 14 \text{ TeV}$, switched on September 10th !

What do we expect?

- test Higgs mechanism
 - SM Higgs boson: $114.4 \text{ GeV} < m_H < 200 \text{ GeV} (!)$
 - $V(H) = \frac{1}{2} M_H^2 H^2 + \lambda_3 H^3 + \lambda_4 H^4$
 - SM: $\lambda_4 = \lambda_3/v = 3 M_H^2/v^2$
- explore physics beyond the Standard Model
 - $\text{SM} \subset$ "Extra Dimensions", "Little Higgs", "Strong interaction" Model
 - $\text{SM} \subset \text{MSSM} \subset \text{SUSY GUT} \subset \text{Supergravity} \subset \text{Superstring} \subset \mathcal{M}\text{-Theory}$
 - BSM something around 1 TeV (?)
- nothing ?!
 - hint of a hidden sector (?)
 - hint of strong interactions in the e.w. sector (?)

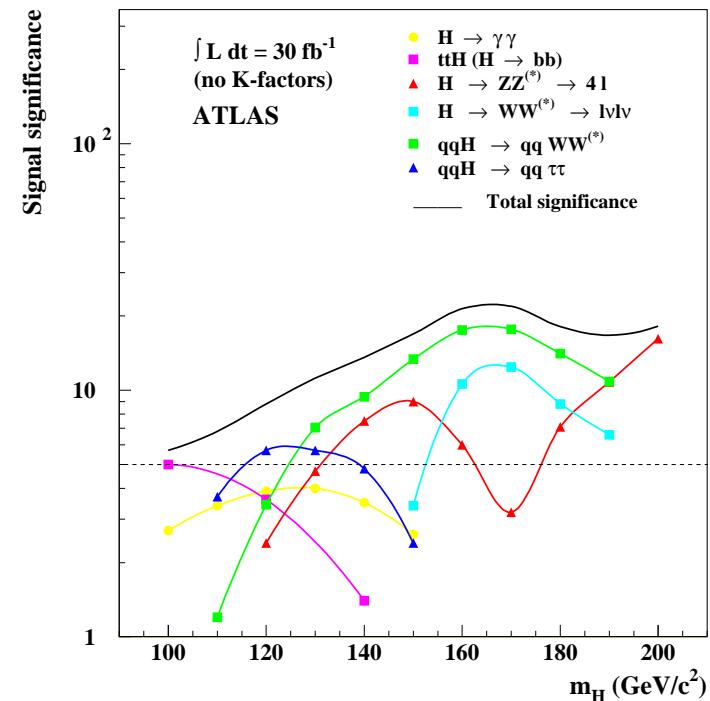


S+B for the Higgs boson



Signal:

- Decays: $H \rightarrow \gamma\gamma$, $H \rightarrow WW^{(*)}$, $H \rightarrow ZZ^{(*)}$, $H \rightarrow \tau^+\tau^-$
- $PP \rightarrow H + 0, 1, 2$ jets Gluon Fusion
- $PP \rightarrow Hjj$ Weak Boson Fusion
- $PP \rightarrow H + t\bar{t}$
- $PP \rightarrow H + W, Z$



S+B for the Higgs boson

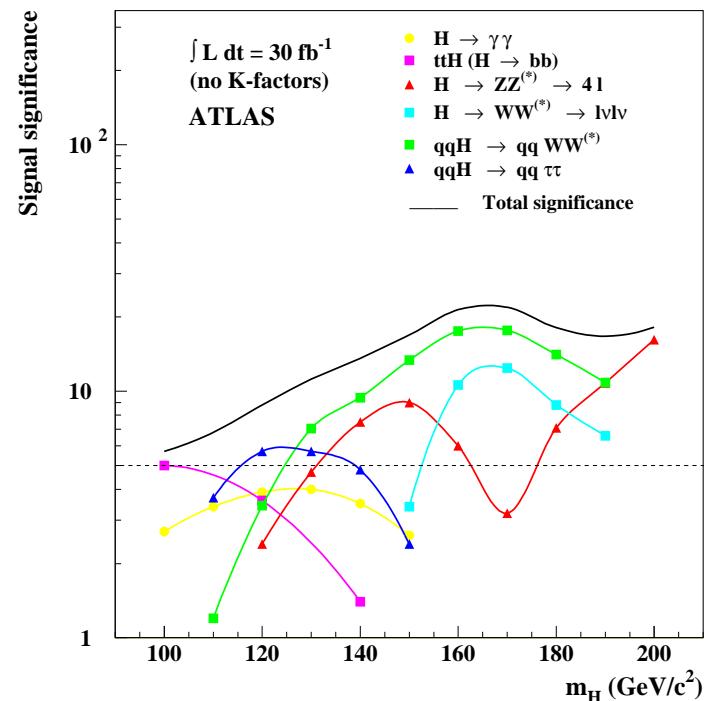


Signal:

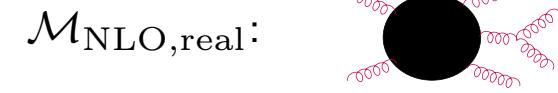
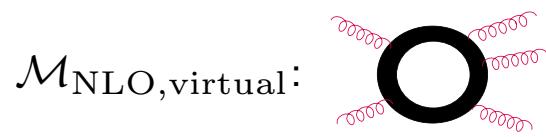
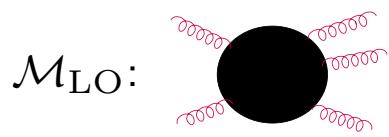
- Decays: $H \rightarrow \gamma\gamma$, $H \rightarrow WW^{(*)}$, $H \rightarrow ZZ^{(*)}$, $H \rightarrow \tau^+\tau^-$
- $PP \rightarrow H + 0, 1, 2$ jets Gluon Fusion
- $PP \rightarrow Hjj$ Weak Boson Fusion
- $PP \rightarrow H + t\bar{t}$
- $PP \rightarrow H + W, Z$

Backgrounds:

- $PP \rightarrow \gamma\gamma + 0, 1, 2$ jets
- $PP \rightarrow WW^*, ZZ^* + 0, 1, 2$ jets
- $PP \rightarrow t\bar{t} + 0, 1, 2$ jets
- $PP \rightarrow V +$ up to 3 jets ($V = \gamma, W, Z$)
- $PP \rightarrow VVV + 0, 1$ jet



Framework for NLO calculations

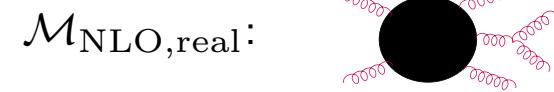
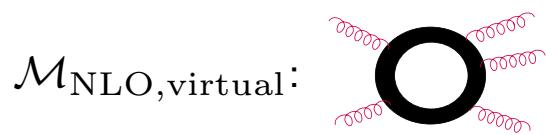
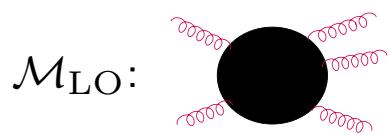


$$\sigma = \sigma_{LO} + \sigma_{NLO}$$

$$\sigma_{LO} = \int dPS_N \frac{1}{2s} \mathcal{O}_N(\{p_j\}) |\mathcal{M}_{\text{LO}}|^2$$

$$\begin{aligned} \sigma_{NLO} = & \int dPS_N \frac{1}{2s} \alpha_s \left(\mathcal{O}_N(\{p_j\}) \left[\mathcal{M}_{\text{LO}} \mathcal{M}_{\text{NLO,V}}^* + \mathcal{M}_{\text{LO}}^* \mathcal{M}_{\text{NLO,V}} \right] \right. \\ & \left. + \int dPS_1 \mathcal{O}_{N+1}(\{p_j\}) |\mathcal{M}_{\text{NLO,R}}|^2 \right) \end{aligned}$$

Framework for NLO calculations



$$\begin{aligned}\sigma &= \sigma_{LO} + \sigma_{NLO} \\ \sigma_{LO} &= \int dPS_N \frac{1}{2s} \mathcal{O}_N(\{p_j\}) |\mathcal{M}_{\text{LO}}|^2 \\ \sigma_{NLO} &= \int dPS_N \frac{1}{2s} \alpha_s \left(\mathcal{O}_N(\{p_j\}) [\mathcal{M}_{\text{LO}} \mathcal{M}_{\text{NLO,V}}^* + \mathcal{M}_{\text{LO}}^* \mathcal{M}_{\text{NLO,V}}] \right. \\ &\quad \left. + \int dPS_1 \mathcal{O}_{N+1}(\{p_j\}) |\mathcal{M}_{\text{NLO,R}}|^2 \right)\end{aligned}$$

- For **IR-safe** observables, $\mathcal{O}_{N+1} \xrightarrow{IR} \mathcal{O}_N$, IR divergences cancel
- IR subtraction: **Frixione, Kunszt, Signer, Soper,...**; dipole method à la **Catani, Seymour** (massless); **Dittmaier, Trocsanyi, Weinzierl, Phaf** (massive).
- automated dipole subtraction: **Gleisberg, Krauss (2007)**; **Seymour, Tevlin (2008)**; **Hasegawa, Moch, Uwer (2008)**; **Frederix, Gehrmann, Greiner (2008)**.
- **Bottleneck**: virtual corrections

Status QCD@NLO for LHC:

$2 \rightarrow 2$: everything you want (see e.g. MCFM by Campbell/Ellis)

Status QCD@NLO for LHC:

$2 \rightarrow 2$: everything you want (see e.g. MCFM by Campbell/Ellis)

$2 \rightarrow 3$: before 2005:

- $pp \rightarrow jjj$, $pp \rightarrow \gamma\gamma j$, $pp \rightarrow Vjj$
- $pp \rightarrow Hjj$ [WBF], $pp \rightarrow Hjj$ [GF], $pp \rightarrow Ht\bar{t}$

after 2005:

- $pp \rightarrow HHH$ (2005)
- $pp \rightarrow VVjj$ [WBF] (2006)
- $pp \rightarrow ZZZ$, $pp \rightarrow t\bar{t}j$, $pp \rightarrow WWj$ (2007)
- $pp \rightarrow VVV$, $pp \rightarrow t\bar{t}Z$ (2008)

Status QCD@NLO for LHC:

$2 \rightarrow 2$: everything you want (see e.g. MCFM by Campbell/Ellis)

$2 \rightarrow 3$: before 2005:

- $pp \rightarrow jjj$, $pp \rightarrow \gamma\gamma j$, $pp \rightarrow Vjj$
- $pp \rightarrow Hjj$ [WBF], $pp \rightarrow Hjj$ [GF], $pp \rightarrow Ht\bar{t}$

after 2005:

- $pp \rightarrow HHH$ (2005)
- $pp \rightarrow VVjj$ [WBF] (2006)
- $pp \rightarrow ZZZ$, $pp \rightarrow t\bar{t}j$, $pp \rightarrow WWj$ (2007)
- $pp \rightarrow VVV$, $pp \rightarrow t\bar{t}Z$ (2008)

$2 \rightarrow 4$: No complete LHC cross section done yet!

- 6 photon amplitude (2007) cut-construction, Feynman diagrams, OPP
- 6 gluon amplitude (1994-2006) cut-construction, ...
- $N > 6$ gluon amplitudes evaluated (2008)
- $q\bar{q} \rightarrow b\bar{b}t\bar{t}$ Bredenstein, Denner, Dittmaier, Pozzorini (2008)
- automation in progress: Rocket Giele, Zanderighi (2008)
Blackhat → talks Forde/Ita, CutTools/OPP → talk Papadopoulos,
“numerical unitarity” → talk Kunszt

The GOLEM project

General One Loop Evaluator for Matrix elements

- Evaluation of 1-loop amplitudes bottleneck for LHC@NLO
- Combinatorial complexity \leftrightarrow numerical instabilities
 \Rightarrow switching between algebraic/numerical representations
- **Aim:** Automated evaluation of numerically stable one-loop amplitudes for multi-leg processes

The GOLEM project

General One Loop Evaluator for Matrix elements

- Evaluation of 1-loop amplitudes bottleneck for LHC@NLO
- Combinatorial complexity \leftrightarrow numerical instabilities
 \Rightarrow switching between algebraic/numerical representations
- **Aim:** Automated evaluation of numerically stable one-loop amplitudes for multi-leg processes
- The GOLEM team: T.B., A. Guffanti, J.Ph. Guillet, G. Heinrich, S. Karg, N. Kauer, E. Pilon, T. Reiter, G. Sanguinetti



Feynman diagrammatic approach:

$$\Gamma^{c,\lambda}(p_j, m_j) = \sum_{\{c_i\}, \alpha} f^{\{c_i\}} \mathcal{G}_\alpha^{\{\lambda\}}$$

$$\mathcal{G}_\alpha^{\{\lambda\}} = \int \frac{d^n k}{i\pi^{n/2}} \frac{\mathcal{N}^{\{\lambda\}}}{D_1 \dots D_N} = \sum_R \mathcal{N}_{\mu_1, \dots, \mu_R}^{\{\lambda\}} I_N^{\mu_1 \dots \mu_R}(p_j, m_j)$$

$$I_N^{\mu_1 \dots \mu_R}(p_j, m_j) = \int \frac{d^n k}{i\pi^{n/2}} \frac{k^{\mu_1} \dots k^{\mu_R}}{D_1 \dots D_N}, \quad D_j = (k - r_j)^2 - m_j^2, \quad r_j = p_1 + \dots + p_j$$

- Passarino-Veltman: $\rightarrow 1/\det(G)^R$, $G_{ij} = 2r_i \cdot r_j$ induce numerical problems
- projection on helicity amplitudes reduces $2k \cdot r_j = D_N - D_j + r_j \cdot r_j$
- Lorentz Tensor Integrals \rightarrow form factor representation à la Davydychev
- Reduction in Feynman parameter space

$$I_N^{\mu_1 \dots \mu_R} = \sum \tau^{\mu_1 \dots \mu_R}(r_{j_1}, \dots, r_{j_r}, g^m) I_N^{n+2m}(j_1, \dots, j_r)$$

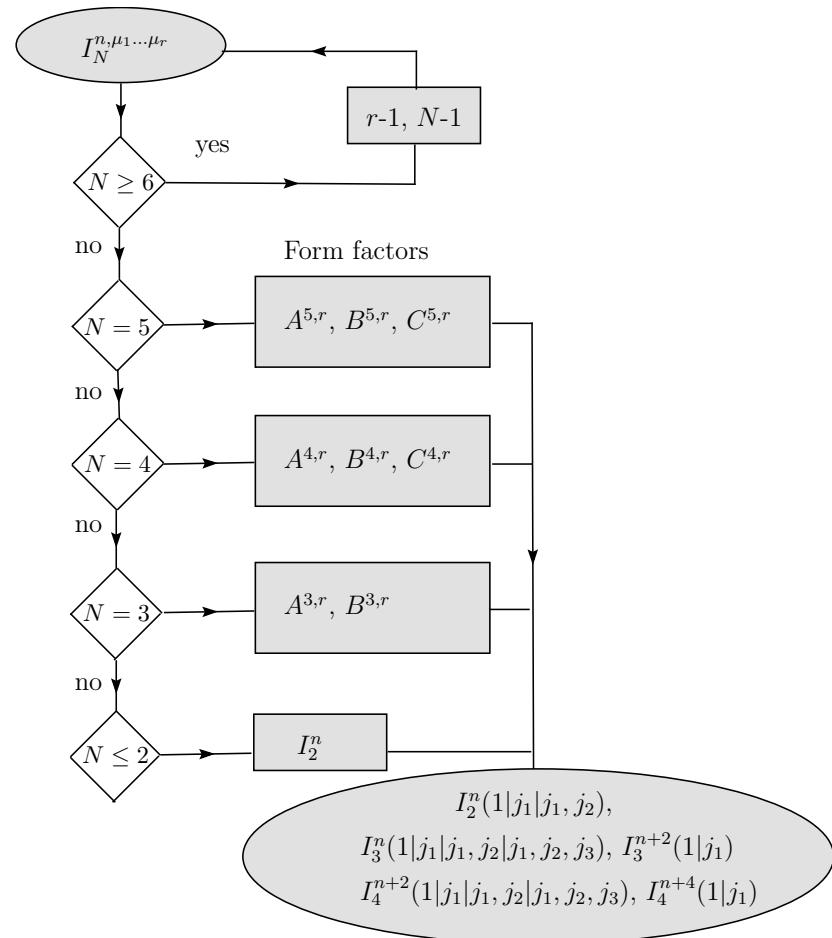
$$I_N^D(j_1, \dots, j_r) = (-1)^N \Gamma(N - \frac{D}{2}) \int_0^\infty d^N z \delta(1 - \sum_{l=1}^N z_l) \frac{z_{j_1} \dots z_{j_r}}{(-\frac{1}{2} z \cdot \mathcal{S} \cdot z)^{N-D/2}}$$

$$\mathcal{S}_{ij} = (r_i - r_j)^2 - m_i^2 - m_j^2$$

Schematic overview of N-point tensor integral reduction

T.B., J.P. Guillet, G. Heinrich (2000); T.B., Guillet, Heinrich, Pilon, Schubert (2005).

- works for general N
- no inverse Gram determinants
- isolation of IR divergences simple
- tractable expressions
- form factors for $N \leq 6$ implemented in Fortran95 code "golem95"
- optional reduction to scalar integrals
- evaluation of rational terms



$$I_{N=3,4}^{n,n+2}(j_1, \dots, j_r) \sim \int_0^1 \prod_{i=1}^4 dz_i \delta(1 - \sum_{l=1}^4 z_l) \frac{z_{j_1} \dots z_{j_r}}{(-\frac{1}{2} z \cdot \mathcal{S} \cdot z - i\delta)^{3-n/2}}$$

Implementation of the algorithm in a nutshell

Preparation:

- Diagram generation: [QGRAF](#) P. Nogueira, [FeynArts 3.2](#) T. Hahn
- Perform colour algebra
- Determine integral basis
- Projection on helicity amplitudes

Implementation of the algorithm in a nutshell

Preparation:

- Diagram generation: **QGRAF** P. Nogueira, **FeynArts 3.2** T. Hahn
- Perform colour algebra
- Determine integral basis
- Projection on helicity amplitudes

From here two independent set-ups:

- a) Convert to form factor representation, link to Fortran95 library “**golem95**”
 - $\mathcal{M}^{\{\lambda\}} \rightarrow C_{box}^{ijk} I_4^{n+2,n+4}(x_i x_j x_k) + C_{tri}^{ijk} I_3^{n,n+2}(x_i x_j x_k) + \dots$
 - In numerically critical phase space regions:
 - compile/run code in quadruple precision
 - use one-dimensional integral representations for $I_{N=3,4}^{n+2,n+4}(x_i x_j x_k)$

Implementation of the algorithm in a nutshell

Preparation:

- Diagram generation: **QGRAF** P. Nogueira, **FeynArts 3.2** T. Hahn
- Perform colour algebra
- Determine integral basis
- Projection on helicity amplitudes

From here two independent set-ups:

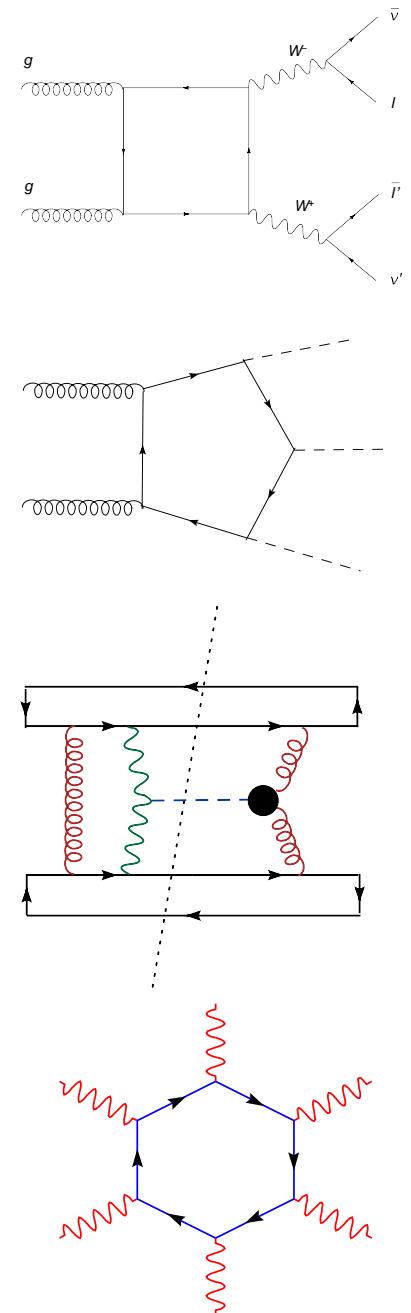
- a) Convert to form factor representation, link to Fortran95 library “**golem95**”
 - $\mathcal{M}^{\{\lambda\}} \rightarrow C_{box}^{ijk} I_4^{n+2,n+4}(x_i x_j x_k) + C_{tri}^{ijk} I_3^{n,n+2}(x_i x_j x_k) + \dots$
 - In numerically critical phase space regions:
 - compile/run code in quadruple precision
 - use one-dimensional integral representations for $I_{N=3,4}^{n+2,n+4}(x_i x_j x_k)$
- b) Symbolic reduction to scalar integrals based on **FORM** and **MAPLE**
 - $\mathcal{M}^{\{\lambda\}} \rightarrow C_{box} I_4^{d=6} + C_{tri} I_3^{d=4-2\epsilon} + C_{bub} I_2^{d=4-2\epsilon} + C_{tad} I_1^{d=4-2\epsilon} + \mathcal{R}$
 - automated method to evaluate \mathcal{R} T.B., Guillet, Heinrich (2006)
 - introduces $1/\det G$ but allows to apply symbolic simplifications

Computations with GOLEM:

Algorithm coded in FORM and FORTRAN 90

some recent applications ...

- $gg \rightarrow W^*W^* \rightarrow l\nu l'\nu'$, GG2WW code
T.B., M. Ciccolini, M. Kramer, N. Kauer (2006)
- $gg \rightarrow HH, HHH$
T.B., S. Karg, N. Kauer, R. Rückl (2006)
- $pp \rightarrow Hjj$ GF/WBF NLO interference $\mathcal{O}(\alpha^2\alpha_s^3)$
J.R. Andersen, T.B., G. Heinrich, J. Smillie (2007)
- $\gamma\gamma \rightarrow \gamma\gamma\gamma\gamma$
T.B., G. Heinrich, T. Gehrmann, P. Mastrolia (2007)



Computations with GOLEM:

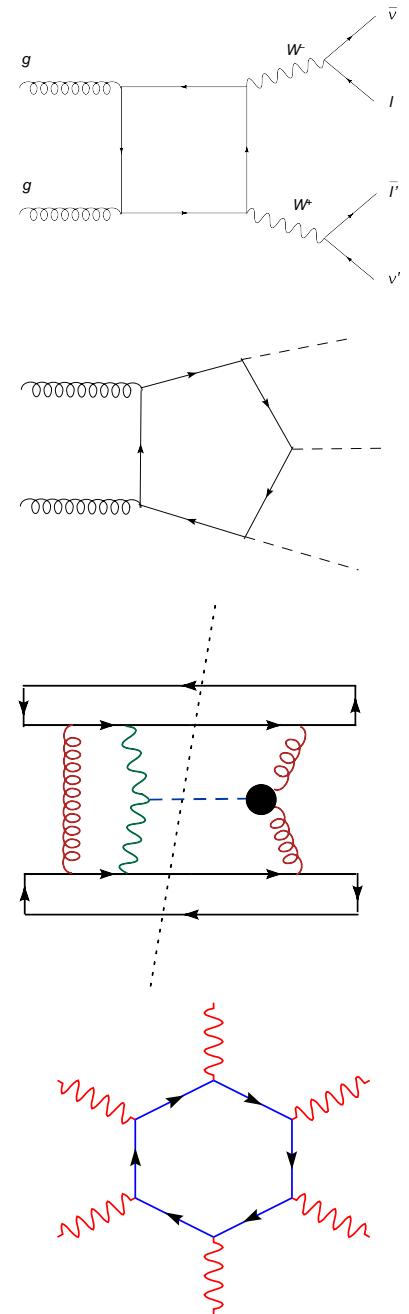
Algorithm coded in FORM and FORTRAN 90

some recent applications ...

- $gg \rightarrow W^*W^* \rightarrow l\nu l'\nu'$, GG2WW code
T.B., M. Ciccolini, M. Kramer, N. Kauer (2006)
- $gg \rightarrow HH, HHH$
T.B., S. Karg, N. Kauer, R. Rückl (2006)
- $pp \rightarrow Hjj$ GF/WBF NLO interference $\mathcal{O}(\alpha^2\alpha_s^3)$
J.R. Andersen, T.B., G. Heinrich, J. Smillie (2007)
- $\gamma\gamma \rightarrow \gamma\gamma\gamma\gamma$
T.B., G. Heinrich, T. Gehrmann, P. Mastrolia (2007)

... and ongoing work:

- $gg \rightarrow Z^*Z^*, \gamma^*Z^*, \gamma^*\gamma^* \rightarrow l\bar{l}'\bar{l}'$, GG2ZZ code
- $pp \rightarrow WWj, ZZj$, $gg \rightarrow WWg, ZZg$
- $pp \rightarrow bbbb$

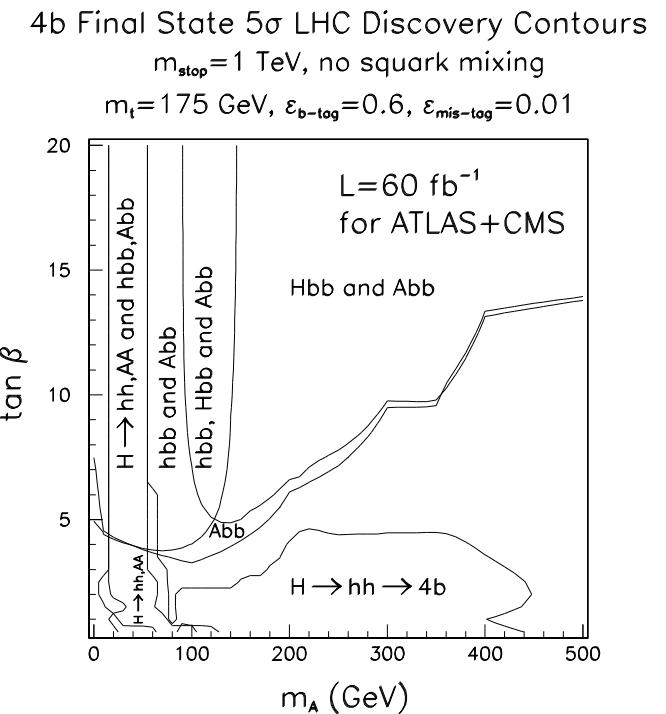


The process $pp \rightarrow b\bar{b}b\bar{b}$ at NLO QCD

Motivation: Higgs search in two Higgs doublet models/MSSM for large $\tan \beta$

Dai, Gunion, Vega 1995/1996; Richter-Was, Froidveaux 1997;
Lafaye, Miller, Muhlleitner, Moretti 2000

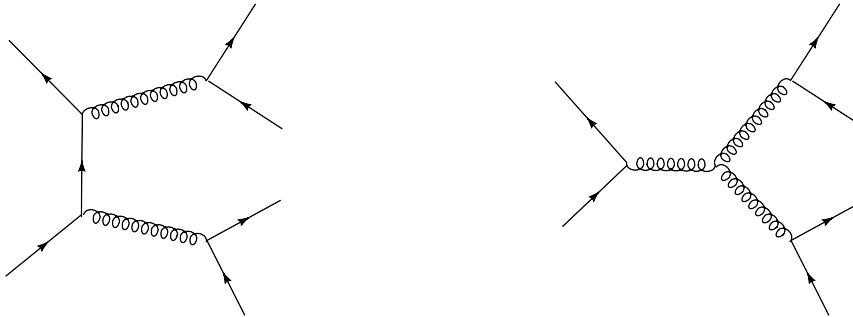
- “ $b\bar{b}\tau^+\tau^-$, $b\bar{b}b\bar{b}$ may provide only access to two of the three neutral Higgs bosons”
- “explicit calculation of K-factors needed.”
- included in the Les Houches wish list



Dai et. al. Phys. Lett. B387 (1996)

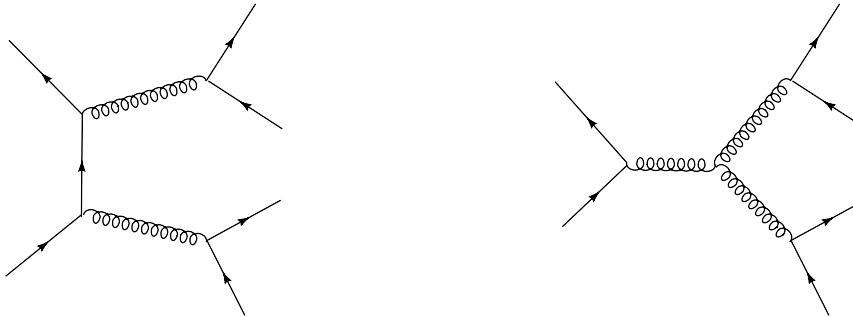
Structure of the amplitude

- 2 initial states: $q\bar{q} \rightarrow b\bar{b}b\bar{b}$, $gg \rightarrow b\bar{b}b\bar{b}$ (under construction)
- $\mathcal{A}(q\bar{q} \rightarrow b_1\bar{b}_2b_3\bar{b}_4) = \mathcal{A}(q\bar{q} \rightarrow b_1\bar{b}_2b'_3\bar{b}'_4) - \mathcal{A}(q\bar{q} \rightarrow b_1\bar{b}'_4b'_3\bar{b}_2)$
- two helicity amplitudes needed: $\mathcal{A}++++++$, $\mathcal{A}++++--$
- six different colour structures: $\mathcal{A} = \sum_{j=1,6} |c_j\rangle \mathcal{A}_j$

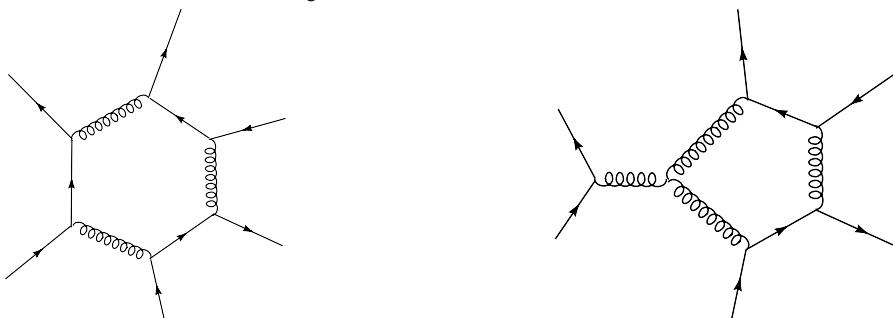


Structure of the amplitude

- 2 initial states: $q\bar{q} \rightarrow b\bar{b}b\bar{b}$, $gg \rightarrow b\bar{b}b\bar{b}$ (under construction)
- $\mathcal{A}(q\bar{q} \rightarrow b_1\bar{b}_2b_3\bar{b}_4) = \mathcal{A}(q\bar{q} \rightarrow b_1\bar{b}_2b'_3\bar{b}'_4) - \mathcal{A}(q\bar{q} \rightarrow b_1\bar{b}'_4b'_3\bar{b}_2)$
- two helicity amplitudes needed: $\mathcal{A}++++++$, $\mathcal{A}++++--$
- six different colour structures: $\mathcal{A} = \sum_{j=1,6} |c_j\rangle \mathcal{A}_j$



- Virtual corrections $q\bar{q} \rightarrow q'\bar{q}'q''\bar{q}''$
~ 250 diagrams, 25 pentagon and 8 hexagon diagrams
- 8 independent scales, $m_b = 0$



Evaluation of loop amplitude

- Diagram generation: [QGRAF](#) P. Nogueira, [FeynArts](#) 3.2 T. Hahn
- Colour algebra [colour flow decomposition] $\Rightarrow \mathcal{A} = \sum_n \sum_c \mathcal{G}_{nc} |c\rangle$

$$\mathcal{G}_{\dots}^{\lambda_1 \lambda_1 \lambda_3 \lambda_3 \lambda_5 \lambda_5} = \sum_{\alpha \beta \gamma} \int \frac{dk^n}{i\pi^{n/2}} T_{\dots}^{\alpha \beta \gamma} \langle 2^{\lambda_1} | \Gamma_{\alpha}^{(1)} | 1^{\lambda_1} \rangle \langle 3^{\lambda_3} | \Gamma_{\beta}^{(2)} | 4^{\lambda_3} \rangle \langle 5^{\lambda_5} | \Gamma_{\gamma}^{(3)} | 6^{\lambda_5} \rangle$$

Evaluation of loop amplitude

- Diagram generation: [QGRAF](#) P. Nogueira, [FeynArts](#) 3.2 T. Hahn
- Colour algebra [colour flow decomposition] $\Rightarrow \mathcal{A} = \sum_n \sum_c \mathcal{G}_{nc} |c\rangle$

$$\mathcal{G}_{\dots}^{\lambda_1 \lambda_1 \lambda_3 \lambda_3 \lambda_5 \lambda_5} = \sum_{\alpha \beta \gamma} \int \frac{dk^n}{i\pi^{n/2}} T_{\dots}^{\alpha \beta \gamma} \langle 2^{\lambda_1} | \Gamma_{\alpha}^{(1)} | 1^{\lambda_1} \rangle \langle 3^{\lambda_3} | \Gamma_{\beta}^{(2)} | 4^{\lambda_3} \rangle \langle 5^{\lambda_5} | \Gamma_{\gamma}^{(3)} | 6^{\lambda_5} \rangle$$

- Helicity projection
e.g. for \mathcal{A}^{++++++} multiply with $\frac{\langle 1^+ | 3| 2^+ \rangle \langle 4^+ | \mu | 3^+ \rangle \langle 6^+ | \mu | 5^+ \rangle}{2[13]\langle 32 \rangle [46]\langle 53 \rangle} = 1$

$$\mathcal{G}_{\dots}^{++++++} = \sum_{\alpha \beta \gamma} \int \frac{dk^n}{i\pi^{n/2}} \frac{T_{\dots}^{\alpha \beta \gamma} \text{tr}^+(132\Gamma_{\alpha}) \text{tr}^+(4\hat{\mu}3\Gamma_{\beta}) \text{tr}^+(6\hat{\mu}5\Gamma_{\gamma})}{2[13]\langle 32 \rangle [46]\langle 53 \rangle}$$

- $\gamma_5 + \text{dim. reg.} \Rightarrow$ 'tHooft-Veltman Scheme and dimension splitting rules
 $k_j = \hat{k}_j, k = \hat{k} + \tilde{k}, \gamma = \hat{\gamma} + \tilde{\gamma}, \{\gamma_5, \hat{\gamma}\} = 0, [\gamma_5, \tilde{\gamma}] = 0$

Evaluation of loop amplitude

Strategy 1: Form factor representation



$$\int \frac{dk^D}{i\pi^{D/2}} \frac{k_1^\mu \dots k_R^\mu}{(k+r_1)^2 \dots (k+r_N)^2} \rightarrow A_N^R, B_N^R, C_N^R \times [g^{\cdot\dots\cdot} r^{\cdot\dots\cdot}]^{\{\mu_1 \dots \mu_R\}}$$

leads to

$$\mathcal{A}^{\{\lambda\}} = \exp(i\phi) \sum \text{FormFactors} \otimes \text{factors}(D) \otimes \prod_j \text{tr}_j^\pm(\{p_l\})$$

Evaluation of loop amplitude

Strategy 1: Form factor representation

-

$$\int \frac{dk^D}{i\pi^{D/2}} \frac{k_1^\mu \dots k_R^\mu}{(k+r_1)^2 \dots (k+r_N)^2} \rightarrow A_N^R, B_N^R, C_N^R \times [g^{\dots} r^{\dots}]^{\{\mu_1 \dots \mu_R\}}$$

leads to

$$\mathcal{A}^{\{\lambda\}} = \exp(i\phi) \sum \text{FormFactors} \otimes \text{factors}(D) \otimes \prod_j \text{tr}_j^{\pm}(\{p_l\})$$

- export to Fortran95 code
- link with `Golem95` library
- all steps are fully automated
- $\text{tr}_j^{\pm}(\{p_l\})$ evaluated numerically

Evaluation of loop amplitude

Strategy 2: Master integral representation

- symbolic evaluation with FORM
- irreducible scalar products canceled algebraically
- at most rank 1 6-point functions, rank 3 5-point functions
- symbolic reduction to master integrals $I_4^{D=6}$, $I_3^{D=4-2\epsilon}$, $I_2^{D=4-2\epsilon}$, \mathcal{R} leads to

$$\mathcal{A}^{\{\lambda\}} = \exp(i\phi) \sum \text{Polynomials}(\{s_{ij}\}) \otimes \text{Master integrals}$$

Evaluation of loop amplitude

Strategy 2: Master integral representation

- symbolic evaluation with FORM
- irreducible scalar products canceled algebraically
- at most rank 1 6-point functions, rank 3 5-point functions
- symbolic reduction to master integrals $I_4^{D=6}$, $I_3^{D=4-2\epsilon}$, $I_2^{D=4-2\epsilon}$, \mathcal{R} leads to

$$\mathcal{A}^{\{\lambda\}} = \exp(i\phi) \sum \text{Polynomials}(\{s_{ij}\}) \otimes \text{Master integrals}$$

- automated simplification of polynomial coefficients with MAPLE
- not as efficient as strategy 1 (room for improvement!)
- used as independent check

Renormalization and IR-structure

$$|\mathcal{A}_{LO+V}\rangle = |\mathcal{A}_0\rangle + \alpha_s |\mathcal{A}_1\rangle = \sum_l \left(|\mathcal{A}_{0,l}\rangle + \alpha_s |\mathcal{A}_{1,l}\rangle \right) \times |c_l\rangle$$

Renormalization and IR-structure

$$|\mathcal{A}_{LO+V}\rangle = |\mathcal{A}_0\rangle + \alpha_s |\mathcal{A}_1\rangle = \sum_l \left(|\mathcal{A}_{0,l}\rangle + \alpha_s |\mathcal{A}_{1,l}\rangle \right) \times |c_l\rangle$$

$$\begin{aligned} \langle \mathcal{A}_{LO+V} | \mathcal{A}_{LO+V} \rangle &= \sum_{kl} \langle \mathcal{A}_{0,k} | \mathcal{A}_{0,l} \rangle \langle c_l | c_k \rangle \left(1 - \frac{1}{\epsilon} \frac{2\beta_0}{\pi} \alpha_s \right) \\ &\quad + \alpha_s \sum_{kl} (\langle \mathcal{A}_{0,k} | \mathcal{A}_{1,l} \rangle + \langle \mathcal{A}_{1,k} | \mathcal{A}_{0,l} \rangle) \langle c_l | c_k \rangle \\ &\quad - \alpha_s \sum_{kl} \langle \mathcal{A}_{0,k} | \mathcal{A}_{0,l} \rangle \langle c_l | \mathbf{I}(\epsilon) | c_k \rangle \end{aligned}$$

$$\begin{aligned} \langle c_j | \mathbf{I}(\epsilon) | c_k \rangle &= \frac{1}{2\pi} \frac{1}{\Gamma(1-\epsilon)} \frac{\mathcal{V}_q}{C_F} \sum_{I,J} \langle c_j | \mathbf{T}_I \cdot \mathbf{T}_J | c_k \rangle \left(\frac{4\pi\mu^2}{2p_I \cdot p_J} \right)^\epsilon \\ \mathcal{V}_q &= C_F \left(\frac{1}{\epsilon^2} - \log^2(\alpha) + \frac{3}{2} \frac{1}{\epsilon} + \frac{3}{2} (\alpha - 1 - \log(\alpha)) + 5 - \frac{\pi^2}{2} \right) \end{aligned}$$

Renormalization and IR-structure

$$|\mathcal{A}_{LO+V}\rangle = |\mathcal{A}_0\rangle + \alpha_s |\mathcal{A}_1\rangle = \sum_l \left(|\mathcal{A}_{0,l}\rangle + \alpha_s |\mathcal{A}_{1,l}\rangle \right) \times |c_l\rangle$$

$$\begin{aligned} \langle \mathcal{A}_{LO+V} | \mathcal{A}_{LO+V} \rangle &= \sum_{kl} \langle \mathcal{A}_{0,k} | \mathcal{A}_{0,l} \rangle \langle c_l | c_k \rangle \left(1 - \frac{1}{\epsilon} \frac{2\beta_0}{\pi} \alpha_s \right) \\ &\quad + \alpha_s \sum_{kl} (\langle \mathcal{A}_{0,k} | \mathcal{A}_{1,l} \rangle + \langle \mathcal{A}_{1,k} | \mathcal{A}_{0,l} \rangle) \langle c_l | c_k \rangle \\ &\quad - \alpha_s \sum_{kl} \langle \mathcal{A}_{0,k} | \mathcal{A}_{0,l} \rangle \langle c_l | \mathbf{I}(\epsilon) | c_k \rangle \end{aligned}$$

$$\begin{aligned} \langle c_j | \mathbf{I}(\epsilon) | c_k \rangle &= \frac{1}{2\pi} \frac{1}{\Gamma(1-\epsilon)} \frac{\mathcal{V}_q}{C_F} \sum_{I,J} \langle c_j | \mathbf{T}_I \cdot \mathbf{T}_J | c_k \rangle \left(\frac{4\pi\mu^2}{2p_I \cdot p_J} \right)^\epsilon \\ \mathcal{V}_q &= C_F \left(\frac{1}{\epsilon^2} - \log^2(\alpha) + \frac{3}{2} \frac{1}{\epsilon} + \frac{3}{2} (\alpha - 1 - \log(\alpha)) + 5 - \frac{\pi^2}{2} \right) \end{aligned}$$

- cancellation of pole parts non-trivial check of implementation
- **Golem** aims to provide the finite combination $|\mathcal{A}_{LO+V}|^2$
- we use “ α ”-improved dipole subtraction method (**Nagy, Trocsanyi**), $\alpha = 0.1$

GOLEM integration strategy

- imperfect numerical cancellations lead to inaccuracies
- dangerous for adaptive integration methods
- ⇒ avoid direct integration of virtual corrections !

GOLEM integration strategy

- imperfect numerical cancellations lead to inaccuracies
- dangerous for adaptive integration methods
- \Rightarrow avoid direct integration of virtual corrections !

Step 1:

- generate unweighted event sample from $\sigma_{LO} \sim |\mathcal{A}_{LO}|^2$
- sort event into histograms

$$\sigma_{LO} = \int d\vec{x} f_0(\vec{x}) = \frac{1}{N} \sum_{j=1}^N f_0(x_j)$$

$$= \sigma_{LO} \int d\vec{y} = \frac{\sigma_{LO}}{N} \sum_{j=1}^N 1$$

$$\langle \mathcal{O} \rangle_{LO} = \frac{\sigma_{LO}}{N} \sum_{j=1}^N \chi(E_j) , \quad \chi(E_j) = \begin{cases} 1, & E_j \in \mathcal{O} \\ 0, & \text{else} \end{cases}$$

GOLEM integration strategy

Step 2:

- reweight each event, E_j , by local K-factor: $K = f_1/f_0$
- no destructive interference with phase space integration !

$$\begin{aligned}\sigma_{LO+virtual} &= \int d\vec{x} f_1(\vec{x}) \\ &= \sigma_{LO} \int d\vec{y} K(\vec{y}), \\ \langle \mathcal{O} \rangle_{LO+virt.} &= \frac{\sigma_{LO}}{N} \sum_{j=1}^N \chi(E_j) K(E_j)\end{aligned}$$

GOLEM integration strategy

Step 2:

- reweight each event, E_j , by local K-factor: $K = f_1/f_0$
- no destructive interference with phase space integration !

$$\begin{aligned}\sigma_{LO+virtual} &= \int d\vec{x} f_1(\vec{x}) \\ &= \sigma_{LO} \int d\vec{y} K(\vec{y}), \\ \langle \mathcal{O} \rangle_{LO+virt.} &= \frac{\sigma_{LO}}{N} \sum_{j=1}^N \chi(E_j) K(E_j)\end{aligned}$$

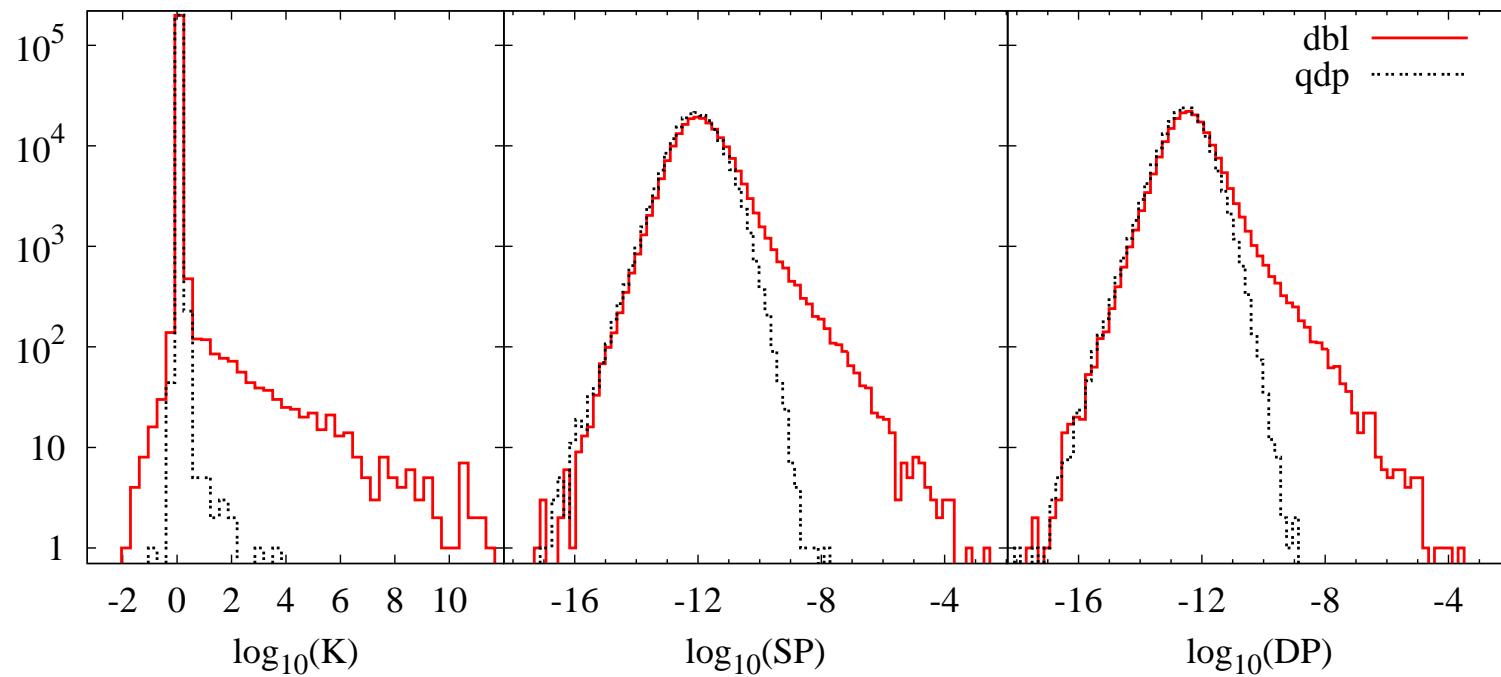
- GOLEM: UV/IR subtracted one-loop amplitudes $\Rightarrow K(E_j)$

For real corrections use:

- LO matrix element/event generators MadGraph, MadEvent, Whizard, Sherpa,...
- ... including dipole subtraction, e.g. MadDipole, TeVJet, Sherpa,...

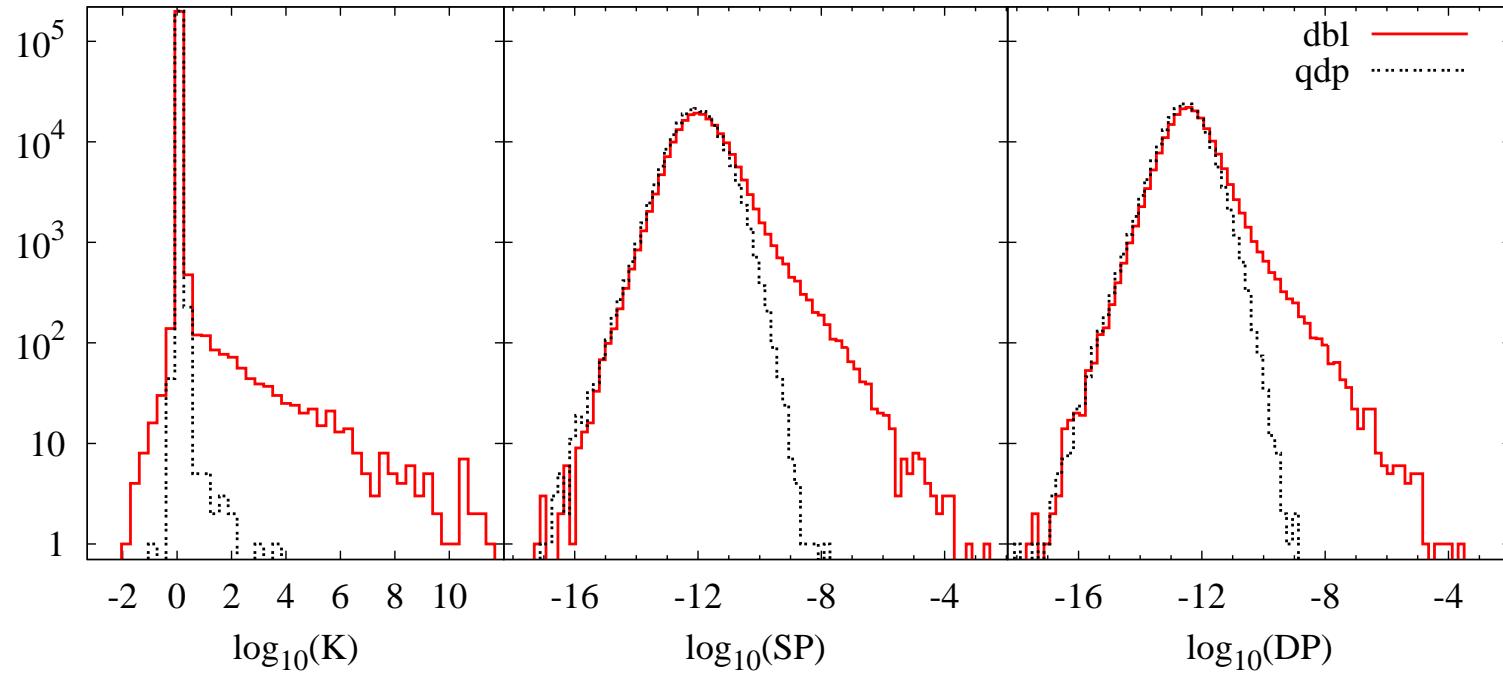
Numerical precision of virtual correction evaluation

- evaluation of 200.000 random phase space points
- cuts: $\eta < |2.5|$, $\Delta R > 0.4$, $p_T > 25$ GeV



Numerical precision of virtual correction evaluation

- evaluation of 200.000 random phase space points
- cuts: $\eta < |2.5|$, $\Delta R > 0.4$, $p_T > 25 \text{ GeV}$



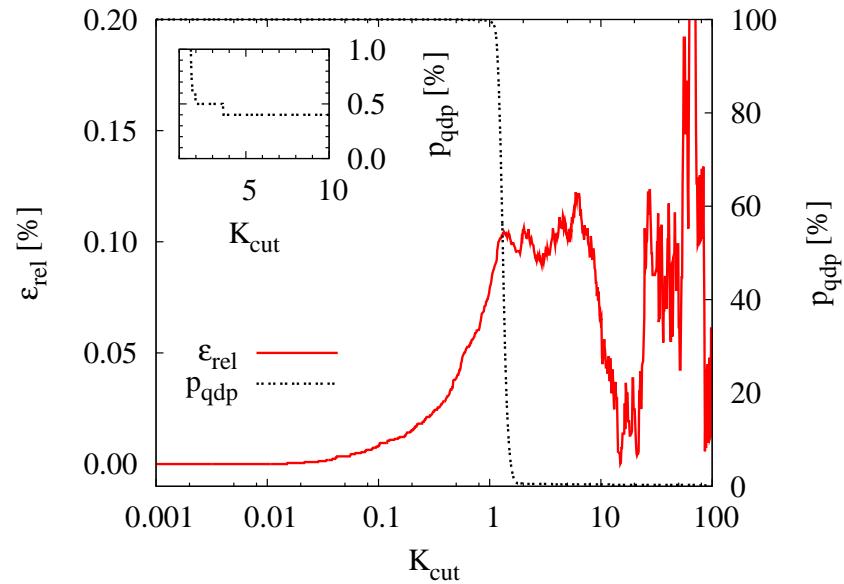
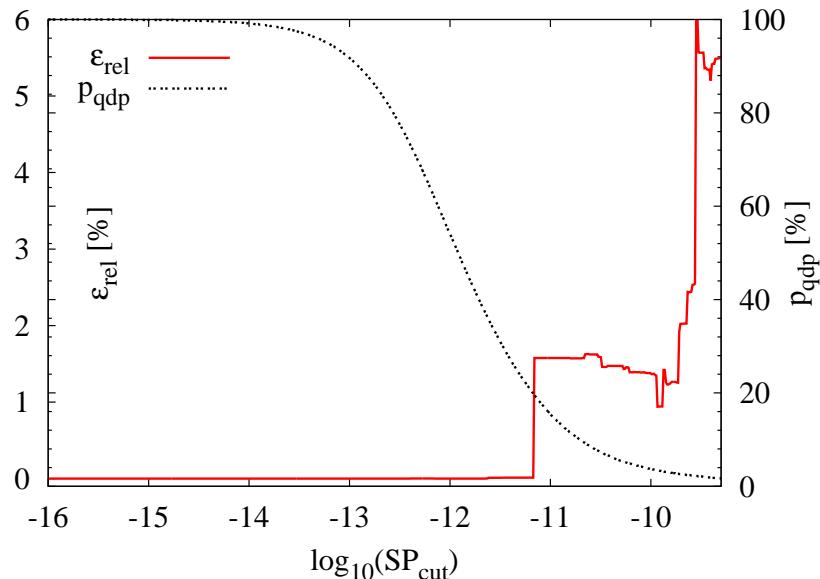
- accuracy of single/double pole cancellation indicator ...
- ...or size of local K-factor indicator for numerical problem
- double scattering singularities: $\det \mathcal{S} \sim p_{T b\bar{b}} \rightarrow 0$
large numerical compensations between diagrams (and also Master integrals !).

Criterion for double/quad precision

Switch to quadruple precision if

- single pole (SP) cancellation better than SP_{CUT}
- K factor is larger than K_{CUT}

$$\epsilon_{rel} = \frac{\sigma(SP/K-cut) - \sigma(quad.prec)}{\sigma(quad.prec)}$$

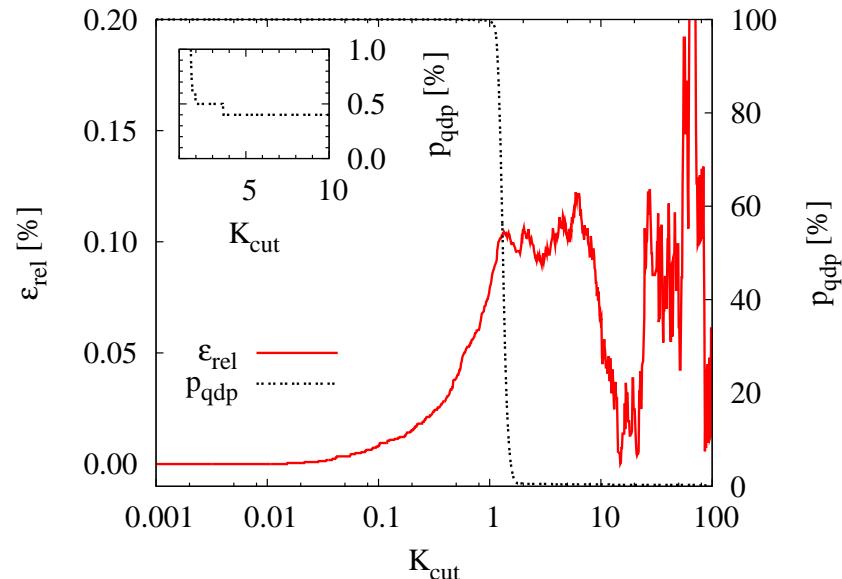
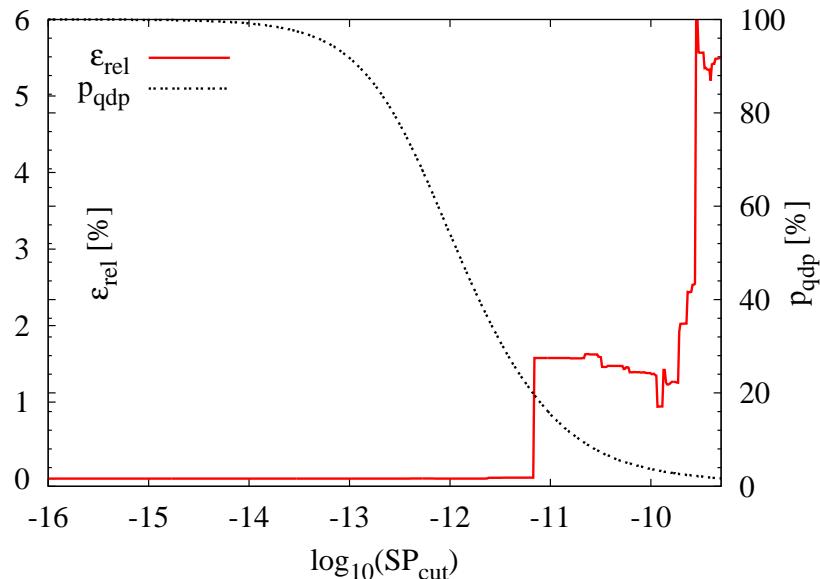


Criterion for double/quad precision

Switch to quadruple precision if

- single pole (SP) cancellation better than SP_{CUT}
- K factor is larger than K_{CUT}

$$\epsilon_{rel} = \frac{\sigma(SP/K-cut) - \sigma(quad.prec)}{\sigma(quad.prec)}$$

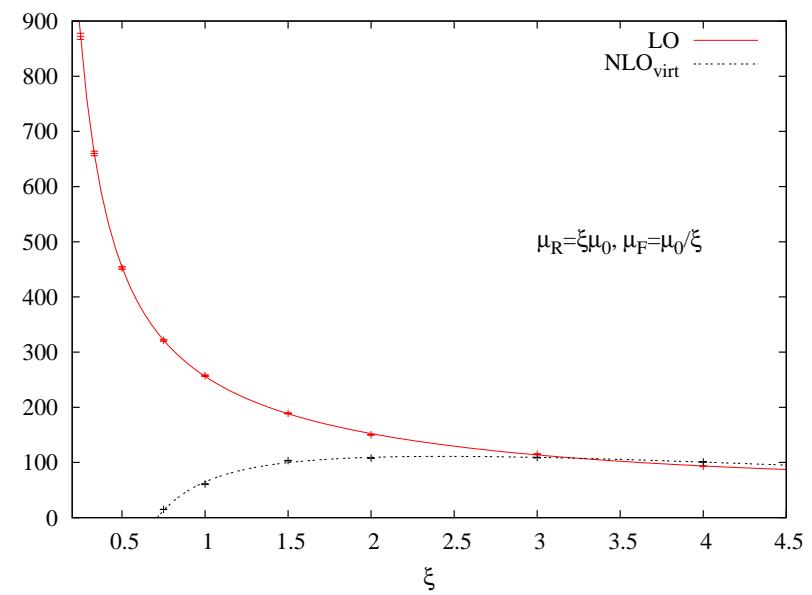
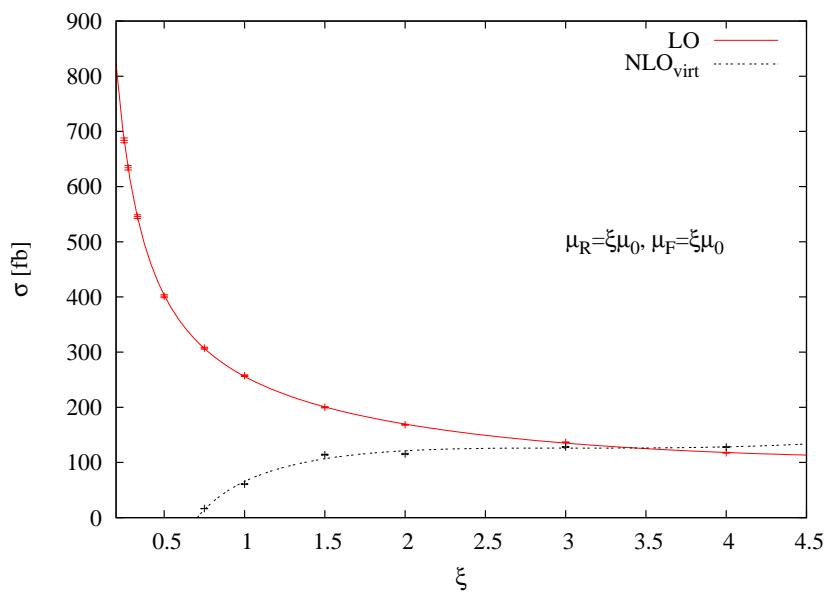


Statistical precision better than 1 % needs:

- SP criterion: needs 20 % of points in quadruple precision
- K-factor criterion: needs less than 1% !

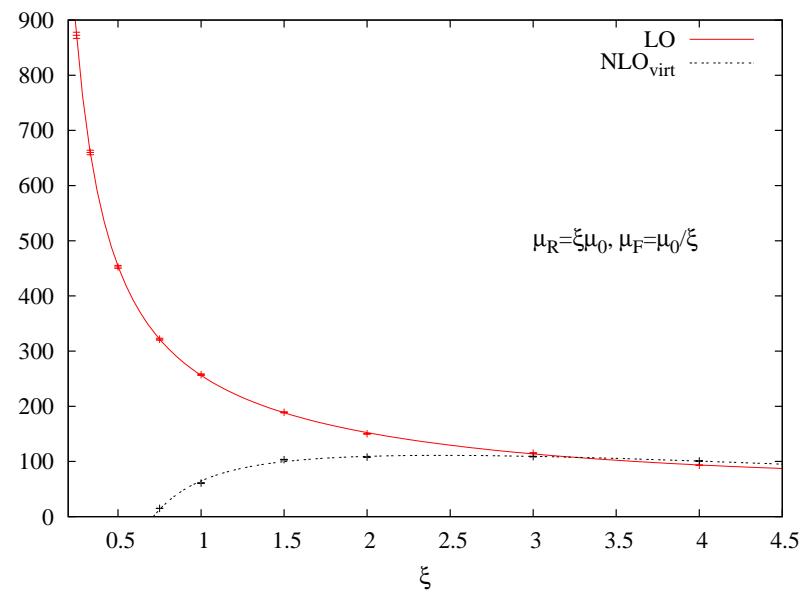
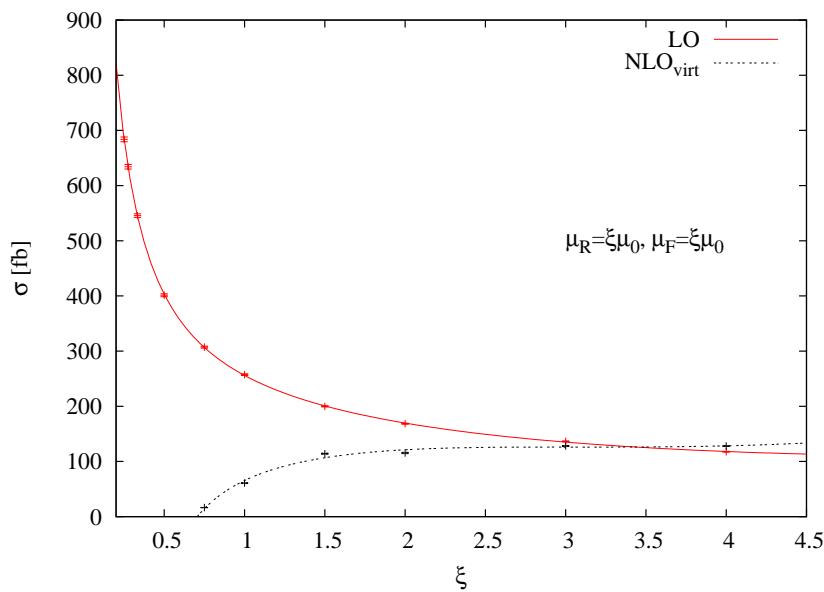
Scale variations

Standard scale choice: $\mu_R = \mu_F = \sum_{j=1}^4 p_T j / 4$



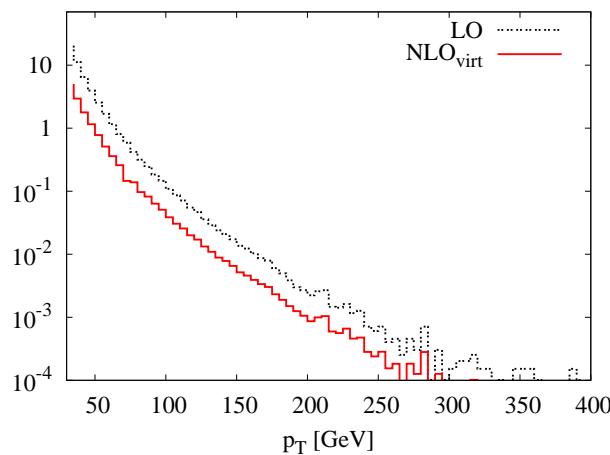
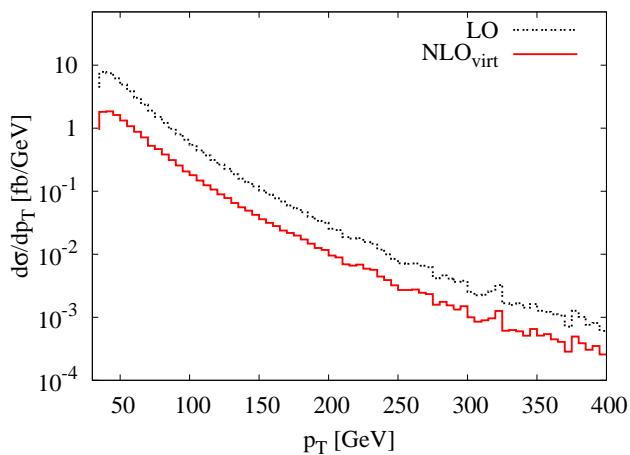
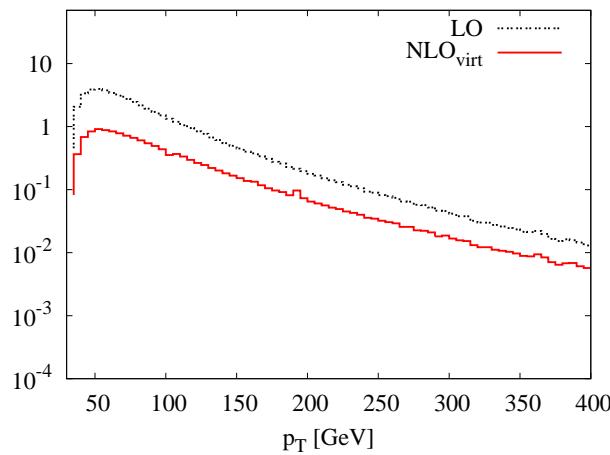
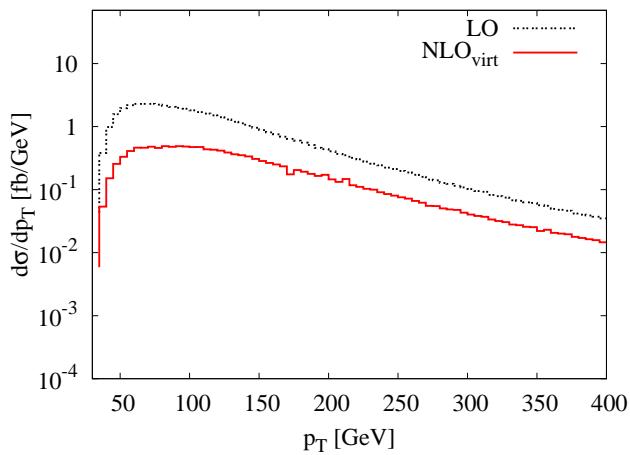
Scale variations

Standard scale choice: $\mu_R = \mu_F = \sum_{j=1}^4 p_T j / 4$

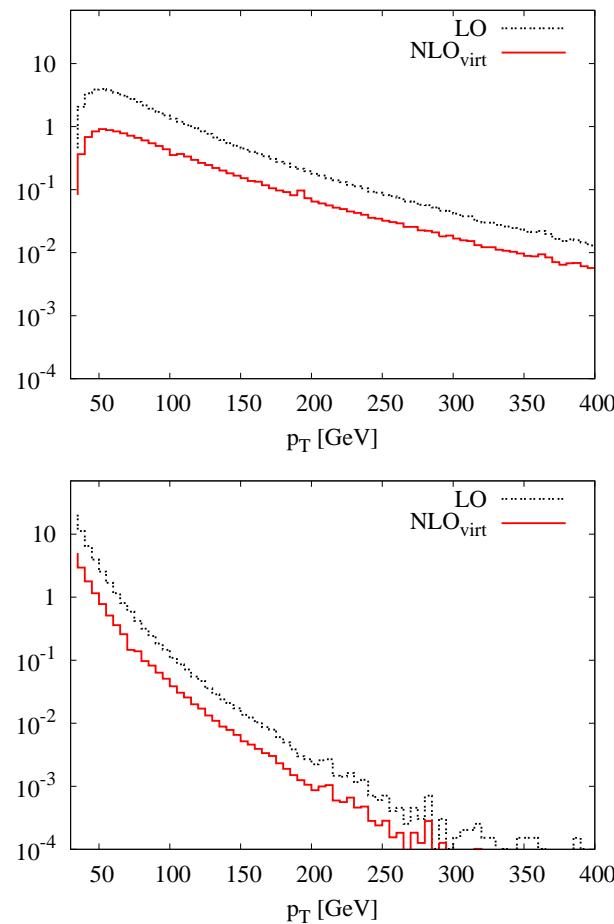
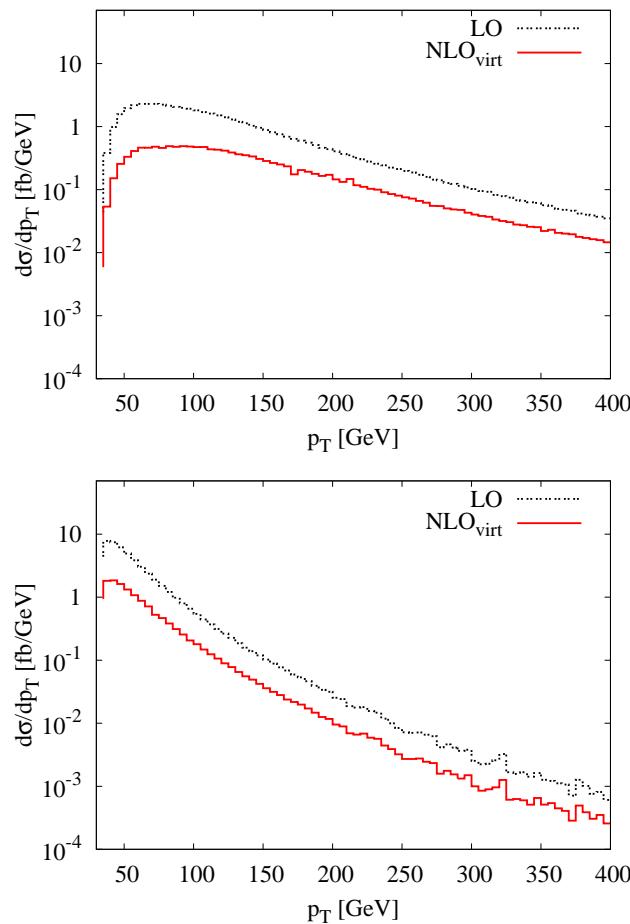


- Dipole subtraction done only in relevant part of phase space ([Nagy, Trocsanyi](#))
→ $\alpha = 0.1$
- real emission contribution not yet added, further compensation of $\alpha_s \log(\mu_F)$
- under construction using [Whizard](#), [MadDipole](#),
implementation ready, amplitudes and dipoles compared

p_T distributions

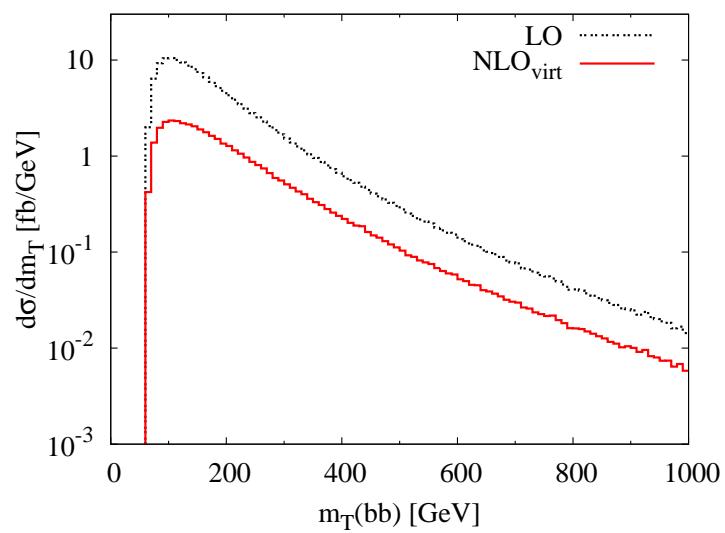
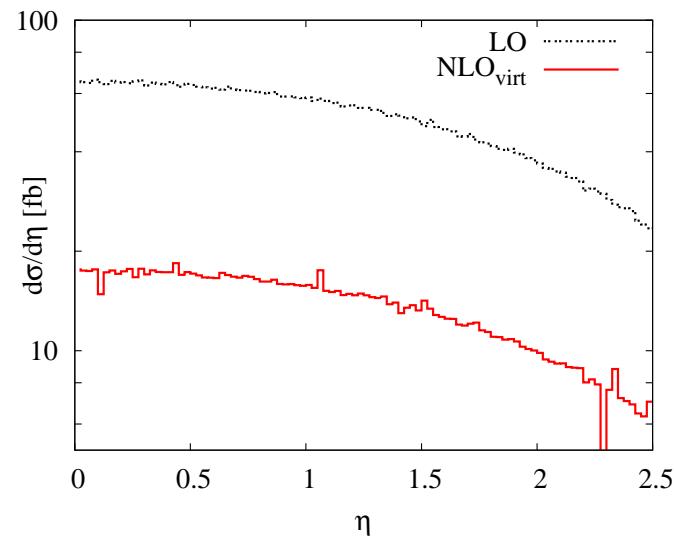


p_T distributions

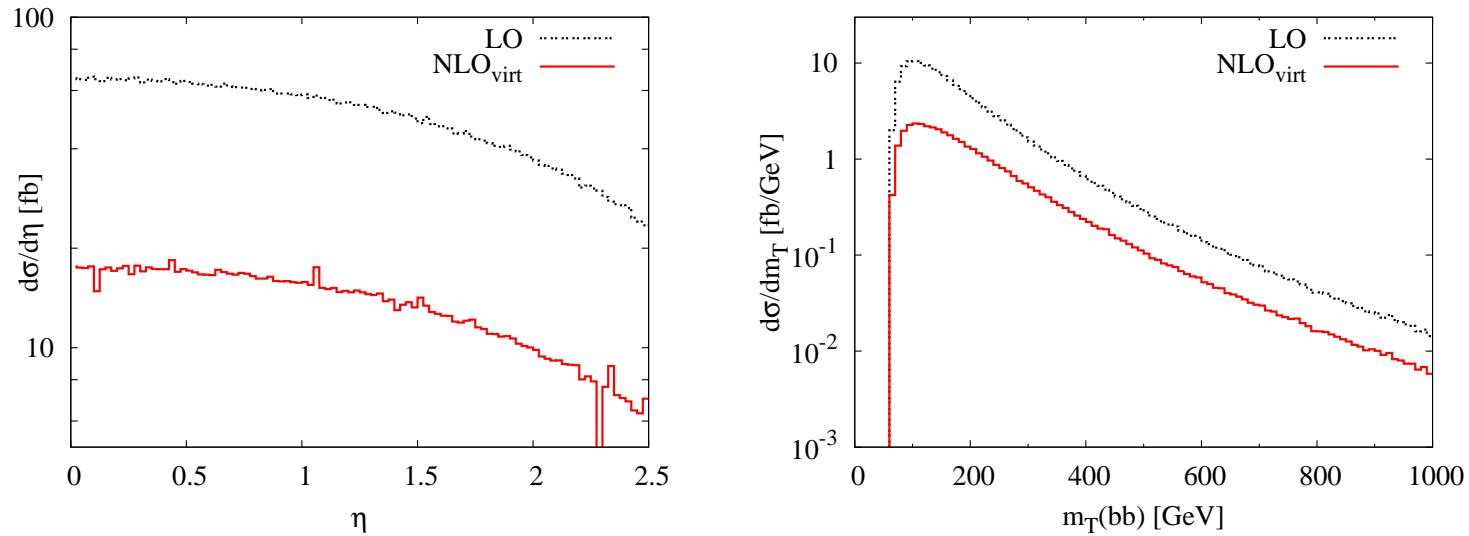


- Distributions obtained by binning histograms from event Ntuples
- evaluate LO with NLO parton distribution functions (here: CTEQ 6.5)
- **Golem** takes care for NLO reweighting
- real emission contribution not yet included

η - and $m_{T\,bb}$ distributions



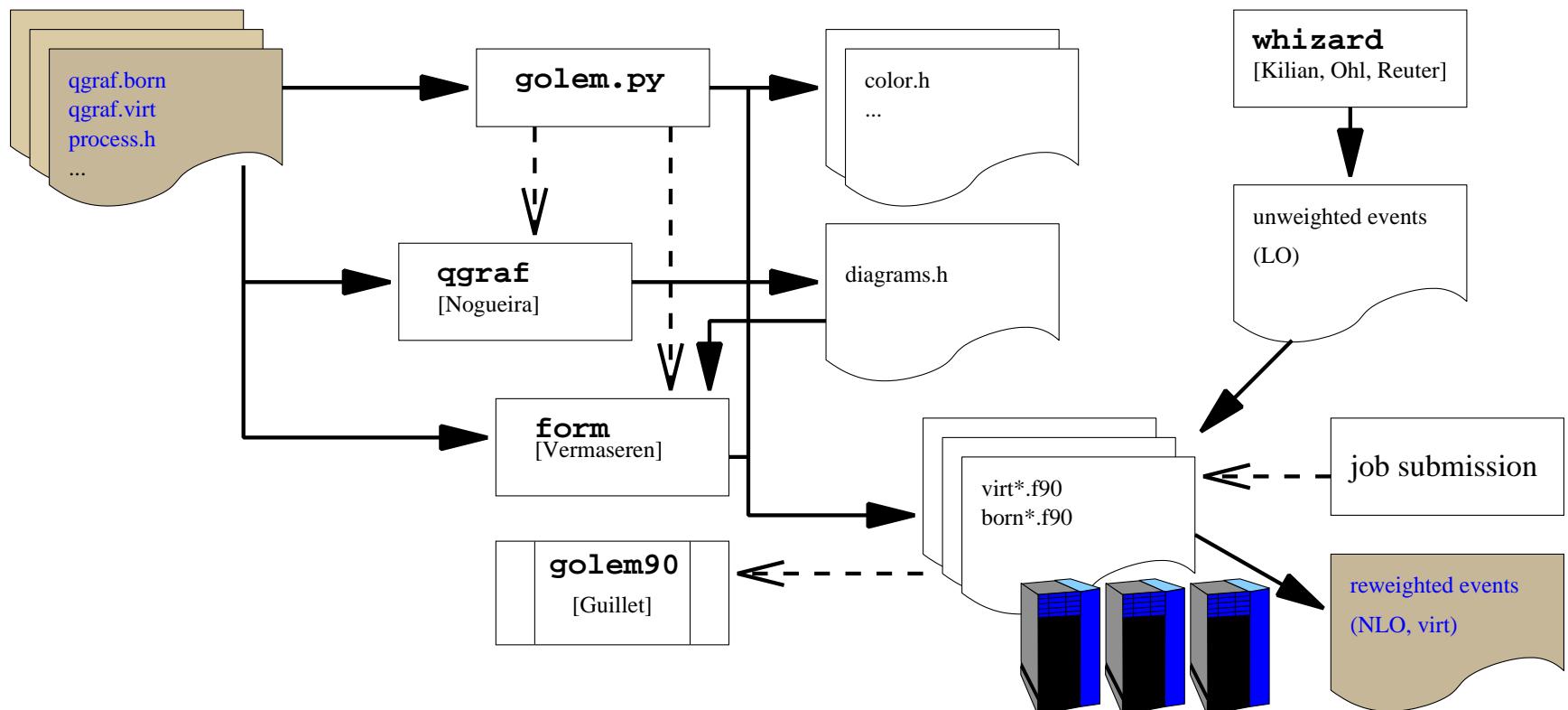
η - and $m_{T\,bb}$ distributions



Philosophy and vision: no standalone NLO computations but rather...

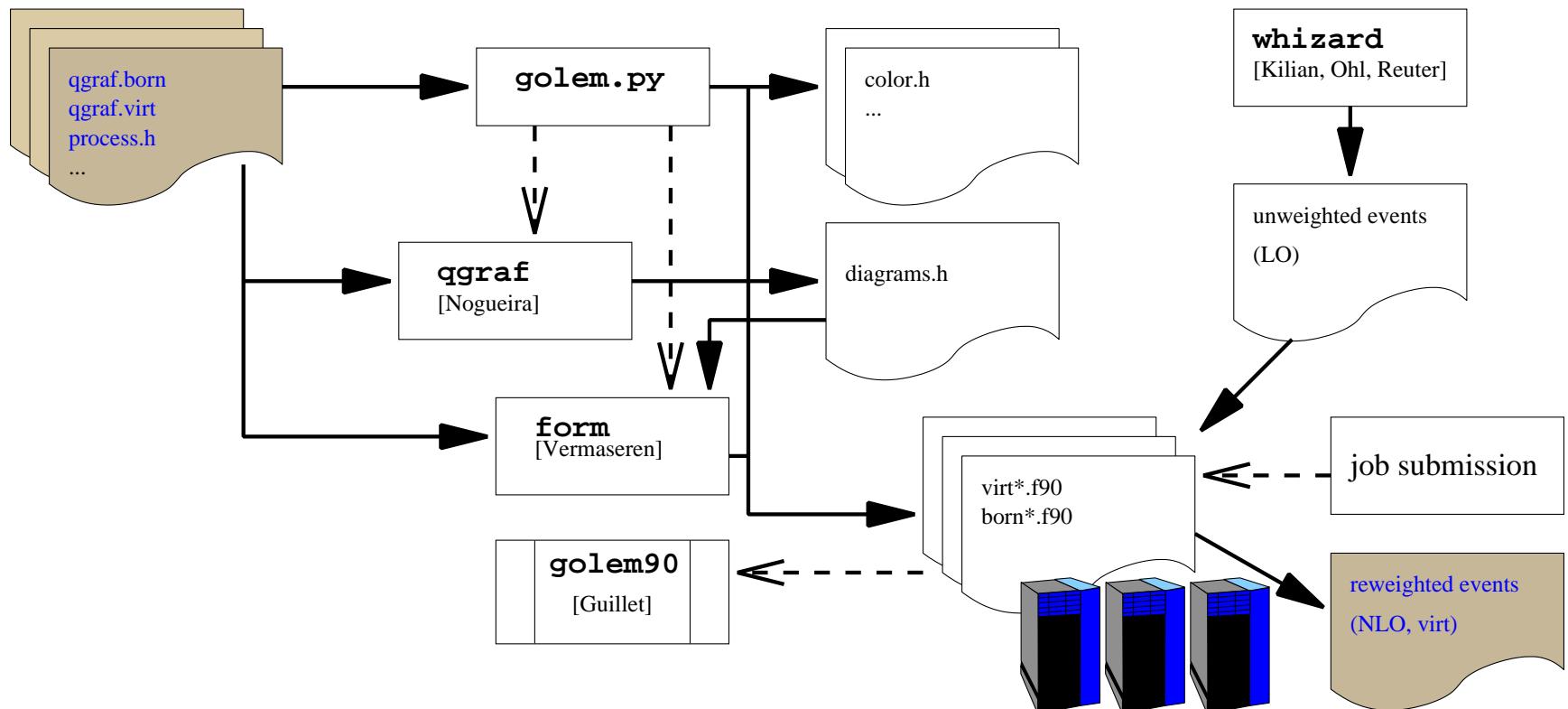
- use Monte Carlo ME event generator including dipole subtraction shapes of distributions o.k. but not normalisation
- data base with all relevant one-loop matrix-elements in one format
- reweighting of LO events can be done on the GRID directly by experimentalists
- merging with parton showers possible (\rightarrow talk of Skands)

Flow chart for evaluating virtual NLO corrections



(from Thomas Reiter's PhD thesis)

Flow chart for evaluating virtual NLO corrections



(from Thomas Reiter's PhD thesis)

- T. Reiter: “performance debate is overrated”, “problem is embarrassingly parallel”
- reweighting of events done in parallel on Edinburgh (ECDF) cluster
- General set-up for NLO computations, to be used for other processes

Summary

LHC phenomenology should be done at the NLO level

Summary

LHC phenomenology should be done at the NLO level

NLO multileg processes still challenging

- lots of activity: algebraic, numerical, unitarity based...
- ... but still no complete $2 \rightarrow 4$ process

Summary

LHC phenomenology should be done at the NLO level

NLO multileg processes still challenging

- lots of activity: algebraic, numerical, unitarity based...
- ... but still no complete $2 \rightarrow 4$ process

GOLEM assaults 1-loop multi-leg processes

- several predictions $2 \rightarrow 2$, $2 \rightarrow 3$ processes
- on the Hexagon front: virtual NLO part of $q\bar{q} \rightarrow b\bar{b}b\bar{b}$ done
- Form factor library public: [golem95-v1.0](#)
- sampling over phase space strategy: reweighting instead of integrating
- set-up highly automated, more to come ...

Summary

LHC phenomenology should be done at the NLO level

NLO multileg processes still challenging

- lots of activity: algebraic, numerical, unitarity based...
- ... but still no complete $2 \rightarrow 4$ process

GOLEM assaults 1-loop multi-leg processes

- several predictions $2 \rightarrow 2$, $2 \rightarrow 3$ processes
- on the Hexagon front: virtual NLO part of $q\bar{q} \rightarrow b\bar{b}b\bar{b}$ done
- Form factor library public: [golem95-v1.0](#)
- sampling over phase space strategy: reweighting instead of integrating
- set-up highly automated, more to come ...

LHC = Long and Hard Calculations ...

- ...but the future is bright !

